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LEONARDO THINKS

Opinion: Visual Mathematics: A New Era?

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<http://www.jstor.org/stable/1575842>.

On 12 October 1492, after a long trip, Christopher Columbus landed on an island of the Americas. On that same day, Italian Renaissance artist Piero della Francesca died. Giorgio Vasari (1511-1574) wrote the following about della Francesca in *The Lives of the Artist*: "He was regarded as a great master of the problems of regular bodies, both arithmetical and geometrical, but he was prevented by the blindness that overtook him in his old age, and then by death, from making known his brilliant researches and the many books he had written, which are still preserved in Borgo San Sepolcro, his native town" [1]. George Bull, editor of the English edition of Vasari's book, adds in a note: "Piero based the relationships in his painting on the laws of Euclidean geometry. The regular bodies were theoretically perfect forms, which, it was thought, could provide the artist with certain, or precisely measurable, relationships through which art could reveal and reproduce the order of nature" [2]. Three treatises by della Francesca have come down to us: *Trattato d'abaco* [3]; *Libellus de quinque corporibus regularibus*[4]; and *De prospectiva pingendi* [5]. As Margaret Davis wrote in 1977, even though it is universally agreed that geometry is a central element of della Francesca's painting, his mathematical treatises are known only superficially, if at all, to art historians. "There exists no study of their contents, sources and significance that places them within the context of Piero's own art and mathematics, as well as in the context of the art and mathematics of Piero's time" [6]. Perhaps the situation will change in the near future; celebrations for the anniversary of della Francesca's death will include an exhibition on the mathematical aspects of his works and perhaps the first publication of the Latin version of *De prospectiva pingendi*.

Morris Kline, in *Mathematics in Western Culture*, describes della Francesca as one of the greatest mathematicians of the fifteenth century:

The artist who perfected the science of perspective was Piero della Francesca. This highly intellectual painter had a passion for geometry



and planned all his works mathematically to the last detail. The placement of each figure was calculated so as to be correct in relation to other figures and to the organization of the painting as a whole. He even used geometry forms for parts of the body and objects of dress and he loved smooth curved surfaces and solidity.

Piero's *The Flagellation* [7] is a masterpiece of perspective. The choice of principal vanishing point and the accurate use of the principles of the focused system tie the characters in the rear of the courtyard to those in front, while the objects are all accommodated to the clearly delimited space. The diminution of the black inlays on the marble floor is also precisely calculated. A drawing in Piero's book on perspectives shows the immense labor which went into this painting. Here, as well as in other paintings, Piero used aerial perspective to enhance the impression of depth. The whole painting is so carefully planned that movement is sacrificed to unity of design [8].

This fascinating description of della Francesca's painting is written by a mathematician! *The Flagellation* is one of the paintings I most love. Every summer I spend 2 weeks on the Adriatic Sea, in a small town called Senigallia, founded by the Senonian Gauls in circa 400 B.C. (One of the most famous of della Francesca's paintings is entitled *The Madonna of Senigallia*.) There I take advantage of the opportunity to take a short trip to the nearby town of Urbino, birthplace of Raphael, and look at *The Flagellation* inside the Galleria Nazionale delle Marche, located in the rooms of the Palazzo Ducale. It is a nearly universal opinion that all mathematical study by Renaissance artists deserves to be viewed from the vantage point of artists and mathematicians studying and learning together [9]. "Although the mathematicians often undeniably furnished the lead, this background is as revealing for artists who developed their mathematical skills independently as for those who were trained and encouraged to formulate mathematical inventions by mathematics teachers", according to Davis [10]. Similar concepts are expressed by mathematician Kline: "The artist was even expected to solve problems involving the motion of cannon balls in artillery fire, a task which in those times called for profound mathematical knowledge. It is no exaggeration to state that the Renaissance artist was the best practicing mathematician and that in the fifteenth century he was also the most learned and accomplished theoretical mathematician" [11].

When I first had the idea of a *Visual Mathematics* special Issue of *Leonardo* I, had in mind della Francesca's *The Flagellation*. Perhaps it is excessive to think that today we are seeing a *new Renaissance*—nevertheless, there is, without a doubt, an interest these days in collaboration between artists and mathematicians. One of the main reasons for this is an emergence of a new way of looking at visual representations for mathematicians; in addition, many artists today have profound interest in so-called new technologies based on a visual approach.

In particular, computer-graphics techniques have been used by mathematicians, not only to visualize already-known phenomena but, more interestingly, to understand old problems not yet completely solved. In some specific cases, such techniques have provided new ways of proving results in mathematical research. The computer has become an instrument that allows



experiments in mathematics to be conducted in a sense and a dimension that are wholly new.

R.Hersh's *Some Proposals for Reviving the Philosophy of Mathematics* was published in 1979 [12]; then, 2 years later, the volume *The Mathematical Experience* by P.J. Davis and R. Hersh saw publication [13]. One of the chapters of the volume by Davis and Hersh is entitled "Why Should I Believe a Computer?" [14]. The two authors recalled a rare event that took place in 1976: an announcement of the proof of a theorem in pure mathematics broke into the news columns of the New York Times. The occasion was the proof by K. Appel and W. Haken of the Four- Color Conjecture:

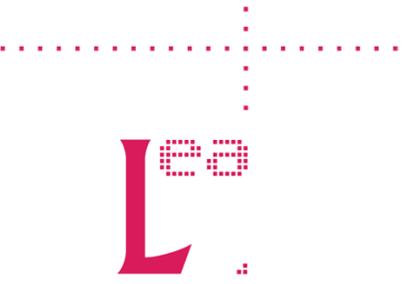
The occasion was newsworthy for two reasons. To begin with, the problem in question was a famous one . . . But the method of proof in itself was newsworthy. For an essential part of the proof consisted of computer calculations. That is to say, the published proof contained computer programs and the output resulting from calculations according to the programs. The intermediate steps by which the programs were executed were of course not published; in this sense the published proofs were permanently and in principle incomplete [15].

According to Davis and Hersh:

In applied mathematics, the computer serves to calculate an approximate answer, when theory is unable to give us an exact answer. . . . But in no way [does] the theory depend on the computer for its conclusions; rather, the two methods, theoretical and mechanical, are like two independent views of the same object; the problem is to coordinate them. . . . The rigorous mathematics of proof remains uncontaminated by the machine . . . In the Haken Appel four-color theorem, the situation is totally different. They present their work as a definitive, complete, rigorous proof. In an expository article on their work, Appel and Haken wrote that most mathematicians who were educated prior to the development of fast computers tend not to think of the computer as a routine tool to be used in conjunction with other and more theoretical tools in advancing mathematical knowledge [16].

Davis and Hersh were thinking of high-speed computers, which have the potential to make thousands of calculations in a short time. At the end of the 1970s, Thomas Banchoff and Charles Strauss of Brown University had the idea of using computer-graphics animation to *visually* investigate the geometrical and topological properties of three-dimensional surfaces. This approach to the use of computers in mathematical research was novel. It became possible to construct a surface on the video terminal and then move it and transform it in order to better understand its properties. It has become a new way of constructing models as well as a guide for intuition.

Computer graphics works not only as a pure visualization of well-known phenomena but also as a new way of studying mathematical problems, in particular, geometrical ones. The great potential of computer graphics as an exploratory medium was recognized by mathematicians soon after the relevant technology became available. As display devices and programming methods grew more sophisticated, so did the depth and scope of applications of



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computer graphics to mathematical problems, as in the case of the discovery of new types of minimal surfaces by William Meeks and David Hoffman. In this instance, the use of computer graphics was essential to obtain a formal proof of the existence of the new surfaces [17].

It is often said that the art of the future depends on new technologies, computer graphics in particular. In considering problems in which visualization plays an important role, mathematicians have obtained images with aesthetic appeal that interest people who are not strictly concerned with the scientific origins of the images. In this way, computers are helping to connect the talents of artists and scientists.

The new branch of mathematics that we call *Visual Mathematics* has been developing in the last few years [18]. The visual use of computers gives rise to new problems for mathematicians: computer graphics could perhaps have a future as the unifying language between art and science. In any case, it is time to propose a comparison between the research of mathematicians and the works of artists, in order to gain an understanding of what interesting results can be expected from the field of Visual Mathematics.



Endnotes

[1] G. Vasari, *Le vite de' pittecellenti Airchitetti Pittori e Scultor italiani da Cimabue insino a'tempi nostri descritte in lingua Toscana, da G. V. Pittore Aretino, con una sua utile e necessaria introduzione alle arti loro* (Florence: 1550); English translation and selection in G. Bull, *The Lives of the Artists* (Harmondsworth, UK: Penguin Books, 1979) p. 191.

[2] Vasari [1].

[3] Piero della Francesca, *Trattato d'Abaco* (Treatise of Abacus) (Florence: Biblioteca Medicea Laurenziana, MS. 280/291-359).

[4] Piero della Francesca, *Libellus de quinque corporibus regularibus* (Libel on the Five Regular Solids) (Rome: Biblioteca Vaticana, MS. Vat. Urb. lat. 632).

[5] Piero della Francesca, *De prospectiva pingendi* (On Painting's Perspective) (Parma: Biblioteca Palatina, MS. 1576) in Italian; (Milan: Biblioteca Ambrosiana, Cod. Ambr. C. 307) in Latin.

[6] M.D. Davis, *Piero della Francesca's Mathematical Treatise* (Ravenna: Longo editore, 1977) p. 1.

[7] See, in this *Visual Mathematics Special Issue of Leonardo*, Color Plate No.3.

[8] M. Kline, *Mathematics in Western Culture* (Harmondsworth, UK: Penguin Books, 1953) p. 166.

[9] See Davis [6] p. 3.

[10] See Davis [6].

[11] See Kline [8] p. 151.

[12] R. Hersh, "Some Proposals for Reviving the Philosophy of Mathematics," *Advances in Mathematics* 1(1979) pp. 31-50.

[13.] P. J. Davis and R. Hersh, *The Mathematical Experience* (Boston: Birkhauser, 1981).

[14] See Davis and Hersh [13] pp. 380-386.

[15] K. Appel and W. Haken, "The Four-Color Problem", in L. A. Steen, ed., *Mathematics Today* (New York: Springer-Verlag, 1978) pp. 153-190.

[16] See Davis and Hersh [13].

[17] M.J. Callahan, D. Hoffman and J. T. Hoffman, "Computer Graphics Tools for the Study of Minimal Surfaces", *Communications of the Association for Computing Machinery (ACM)* 31, No.6, 648-661 (1988).

[18] M. Emmer, *La perfezione visibile: artea matematica* (Rome: Theoria edizioni, 1991).